

Name: _____

Solutions

1. Write 2.5% as a simplified fraction.

$$2.5\% = \frac{2.5}{100} = .025 \text{ as a decimal}$$

↑ tenth
↑ hundredth
↑ thousandth

$$= \frac{25}{1000} = \frac{\cancel{25} \cdot 1}{\cancel{25} \cdot 40} = \frac{1}{40} \text{ as a simplified fraction}$$

2. Simplify the expression
- $2y^2 - 5y + 13 - 13y^2 + 4y - 3$

$$= (2y^2 - 13y^2) + (-5y + 4y) + (13 - 3)$$

$$= \boxed{-11y^2 + y + 10}$$

3. Evaluate the rational expression
- $\frac{2t^3 - 8}{t^2 + 5}$
- for
- $t = -1$
- .

$$= \frac{2(-1)^3 - 8}{(-1)^2 + 5} = \frac{2(-1) - 8}{1 + 5} = \frac{-2 - 8}{6} = \frac{-10}{6}$$

$$= \boxed{-\frac{5}{3}}$$

4. Write the number 127,000 in scientific notation.

$$= \boxed{1.27 \times 10^5}$$

1.27000
1 2 3 4 5

5. Translate the following into an algebraic expression: equation!

The sum of a number and 16, divided by 4, yields the quotient of the number and 9.

$$\boxed{\frac{x + 16}{4} = \frac{x}{9}}$$

6. Find the restricted value(s) for

$$\frac{x^2 - 9}{x^2 - 2x - 15}$$

↑ means "not allowed"

and dividing by zero is not allowed. So set

$$x^2 - 2x - 15 = 0$$

Quadratic so use ZPP

$$(x-5)(x+3) = 0$$

Factor.

set factors = 0

$$x-5=0 \quad \text{or} \quad x+3=0$$

$$x=5 \quad \text{or} \quad x=-3$$

7. 31.5 is what percent of 63?

$$31.5 = p \cdot 63$$

"is" ↔ =

"of" ↔ times

$$\frac{31.5}{63} = \frac{p \cdot 63}{63} \Rightarrow$$

$$p = 0.5 = 50\%$$

(move dec pt 2 places to the right)

8. Simplify and write your answer in scientific notation:

$$(8 \times 10^4) \cdot (2.5 \times 10^{-9})$$

$$= (8 \cdot 2.5) \times (10^4 \cdot 10^{-9})$$

$$= 20 \times 10^{4+(-9)} = 20 \times 10^{-5}$$

$$= (2.0 \times 10^1) \times 10^{-5} = 2.0 \times 10^{-4}$$

But 20 is not between 1 and 10!
So put 20 in SN!
 $20 = 2.0 \times 10^1$

9. If -24 is added to a number, the sum will be 6 times the number. Find the number.

$$-24 + x = 6x$$

-x -x

$$\frac{-24}{5} = \frac{5x}{5} \Rightarrow$$

$$x = \frac{-24}{5} \quad \text{or} \quad -4.8$$

10. Write the slope-intercept form of the line passing through the points (2, 3) and (6, 4).

$$m = \frac{4-3}{6-2} = \frac{1}{4}$$

Use point-slope form:
I used (2, 3). (6, 4) would give the same answer.

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{1}{4}(x - 2)$$

$$y - 3 = \frac{1}{4}x - \frac{2}{4}$$

$$y = \frac{1}{4}x - \frac{1}{2} + \frac{3}{1} \cdot \frac{2}{2}$$

$$y = \frac{1}{4}x + \frac{5}{2}$$

When solving equations, Ask yourself if each equation is **LINEAR, QUADRATIC, OR RATIONAL.**

Solve the following equations:

11. $5x + 4 = 2(x + 8)$ **Linear**

$$\begin{array}{r} 5x + 4 = 2x + 16 \\ -2x \quad -2x \\ \hline 3x + 4 = 16 \\ -4 \quad -4 \\ \hline \end{array}$$

$$\frac{3x}{3} = \frac{12}{3}$$

$$\boxed{x = 4}$$

12. $5.14x + 1.32 = 4.7x$ **Linear**

$$\begin{array}{r} 5.14x + 1.32 = 4.7x \\ -5.14x \quad -5.14x \\ \hline 1.32 = -0.44x \\ -0.44 \quad -0.44 \\ \hline \end{array} \Rightarrow -3 = x \text{ or } \boxed{x = -3}$$

13. $x^2 - 13x + 40 = 0$ **Quadratic**

$$(x - 5)(x - 8) = 0$$

$$x - 5 = 0 \text{ or } x - 8 = 0$$

$$\boxed{x = 5 \text{ or } x = 8}$$

14. $(x - 2)(x + 1) = 28$ **Quadratic**

Need to get 0 on this side to use ZPP

$$\begin{array}{l} x^2 - 2x + 1x - 2 = 28 \\ x^2 - 2x + 1x - 2 - 28 = 0 \end{array} \Rightarrow \begin{array}{l} x^2 - x - 30 = 0 \\ (x - 6)(x + 5) = 0 \end{array} \Rightarrow \begin{array}{l} x - 6 = 0 \text{ or } x + 5 = 0 \\ \boxed{x = 6 \text{ or } x = -5} \end{array}$$

15. $\frac{3}{t+2} = \frac{6}{t+16}$ **Rational** Proportion, so can cross-multiply

$$\begin{array}{r} 3(t+16) = 6(t+2) \\ 3t + 48 = 6t + 12 \\ -3t \quad -3t \\ \hline 48 = 3t + 12 \end{array}$$

$$\begin{array}{r} 48 = 3t + 12 \\ -12 \quad -12 \\ \hline 36 = 3t \\ \frac{36}{3} = \frac{3t}{3} \Rightarrow \boxed{t = 12} \end{array}$$

16. $\frac{7}{3x} - \frac{x+2}{x} = \frac{3}{7}$ **Rational** LCD = $3x \cdot 7 = 21x \Rightarrow$ Multiply each term by LCD

$$\frac{7}{3x} \cdot \frac{7}{1} - \frac{(x+2)}{x} \cdot \frac{7}{1} = \frac{3}{7} \cdot \frac{21x}{1} \Rightarrow 49 - 21(x+2) = 9x$$

$$\Rightarrow 49 - 21x - 42 = 9x \Rightarrow \frac{7}{30} = \frac{30x}{30} \Rightarrow \boxed{x = \frac{7}{30}}$$

Solve each system of linear equations. If a system has no solution or infinitely many solutions, state so.

17. $x + 4y = 18 \Rightarrow x = 18 - 4y$ so $3(18 - 4y) - y = -24$
 $3x - y = -24 \leftarrow 54 - 12y - y = -24$

$$\begin{array}{r} 54 - 13y = -24 \\ \underline{-54} \quad \underline{-84} \\ -13y = -78 \\ \underline{-13} \quad \underline{-13} \\ y = 6 \end{array}$$

so $x = 18 - 4 \cdot 6$
 $x = 18 - 24 = -6$

$(-6, 6)$

18. $y = 3 - 2x$
 $10x + 5y = 11$

$$10x + 5(3 - 2x) = 11$$

$$10x + 15 - 10x = 11$$

$$15 = 11 \quad \text{false}$$

NO SOLUTION

Factor each of the following polynomials completely:

19. $3x^2 + 13x + 14$

Trial & error
 $(3x \quad)(x \quad)$

Factors of 14 are $1 \cdot 14$
 $2 \cdot 7$

\rightarrow we want the Outer & inner to add up to $13x$

$(3x + 7)(x + 2)$

$7x \checkmark$
 $6x \checkmark$

AC Method $A = 3, C = 14$
 $AC = 42$. A pair of factors of 42 that add up to 13 are 6 and 7.

$$3x^2 + 13x + 14 = (3x^2 + 6x) + (7x + 14)$$

Factor by Grouping

$$= 3x(x + 2) + 7(x + 2)$$

$$= (x + 2)(3x + 7)$$

20. $(8x^2 - 3y) + (8xy - 3x)$

4 terms
 Factor by Grouping

NO GCF to pull out

Rearrange:

$$(8x^2 + 8xy) - 3y - 3x$$

$$= (8x^2 + 8xy) + (-3y - 3x)$$

$$= 8x(x + y) - 3(y + x)$$

$$= 8x(x + y) - 3(x + y) = (x + y)(8x - 3)$$

21. $4m^3 - 12m^2 - 40m$

$$= 4m(m^2 - 3m - 10)$$

$$= 4m(m - 5)(m + 2)$$

22. $x^2 - \frac{4}{9}$

$$= (x + \frac{2}{3})(x - \frac{2}{3})$$

$(\frac{2}{3})^2 = \frac{4}{9}$ use $A^2 - B^2 = (A + B)(A - B)$

Perform the indicated operations and simplify. Express your answers using positive exponents only.

23. $(4 \cdot 5)^2 + 4 \cdot 5^2$

Use order of operations.

Parentheses
Exponents
Mult/Div
Add/Sub

$$= (20)^2 + 4 \cdot 25 = 400 + 100 = \boxed{500}$$

24. $\frac{18x^{24}(y^3)^2}{6x^3y^0z^2}$
 $y^0 = 1$

$$= \frac{18 \cdot x^{24} \cdot y^6}{6 \cdot x^3 \cdot 1 \cdot z^2}$$

$$= \frac{3x^{21}y^6}{z^2}$$

25. $\frac{x^2 - 7x}{x^2 + 2x} \cdot \frac{x^2 + 7x + 12}{x^2 - 4x - 21}$

$$= \frac{\cancel{x(x-7)}}{\cancel{x(x+2)}} \cdot \frac{\cancel{(x+3)}(x+4)}{\cancel{(x-7)}(x+3)} = \boxed{\frac{x+4}{x+2}}$$

26. $(2x - 3)^2$

$$= (2x-3)(2x-3)$$

and FOIL it out:

$$= 4x^2 - 6x - 6x + 9$$

$$= \boxed{4x^2 - 12x + 9}$$

← correct

BEWARE!

$$(A-B)^2 \neq A^2 - B^2$$

$$\text{so } (2x-3)^2 \neq 4x^2 - 9 \leftarrow \text{wrong}$$

27. $\frac{a^2 + 5a - 14}{a + 3} \div \frac{a - 2}{a^2 + 2a - 3}$

$$= \frac{a^2 + 5a - 14}{(a+3)} \cdot \frac{a^2 + 2a - 3}{(a-2)}$$

$$= \frac{(a+7)(a-2)}{(a+3)} \cdot \frac{(a+3)(a-1)}{\cancel{(a-2)}}$$

$$= \frac{(a+7)(a-1)}{(a+3)}$$

$$= \frac{(a+7)(a-1)}{(a+3)}$$

$$= \boxed{\begin{matrix} (a+7)(a-1) \\ \text{or} \\ a^2 + 6a - 7 \end{matrix}}$$

$$28. 4\sqrt{18} - \sqrt{72} + 3\sqrt{63} = 4 \cdot 3\sqrt{2} - 6\sqrt{2} + 3 \cdot 3\sqrt{7} = 12\sqrt{2} - 6\sqrt{2} + 9\sqrt{7} = \boxed{6\sqrt{2} + 9\sqrt{7}}$$

$$\sqrt{18} = \sqrt{9 \cdot 2} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$$

$$\sqrt{72} = \sqrt{36 \cdot 2} = \sqrt{36} \sqrt{2} = 6\sqrt{2}$$

$$\sqrt{63} = \sqrt{9 \cdot 7} = \sqrt{9} \cdot \sqrt{7} = 3\sqrt{7}$$

$$29. -2y^2(4xy + 15y^3)$$

$$= \boxed{-8xy^3 - 30y^5}$$

$$30. \frac{3(2x+7)}{2x^2+9x-18} + \frac{x+5}{x+6} = \frac{6x+21}{(2x-3)(x+6)} + \frac{(x+5) \cdot (2x-3)}{(x+6)(2x-3)}$$

$$= \frac{6x+21}{(2x-3)(x+6)} + \frac{2x^2+7x-15}{(2x-3)(x+6)} = \frac{2x^2+13x+6}{(2x-3)(x+6)}$$

$$= \frac{(2x+1)\cancel{(x+6)}}{(2x-3)\cancel{(x+6)}} = \boxed{\frac{2x+1}{2x-3}}$$

$$31. \frac{6a^3 - 10a^2 - 16a}{2a^2} = \frac{6a^3}{2a^2} - \frac{10a^2}{2a^2} - \frac{16a}{2a^2}$$

$$= \boxed{3a - 5 - \frac{8}{a}}$$

$$32. \frac{\left(\frac{k+1}{28k}\right)}{\left(\frac{5k-2}{21k}\right)} = \frac{k+1}{28k} \cdot \frac{21k}{5k-2} = \frac{(k+1)}{\cancel{7} \cdot 4 \cdot \cancel{k}} \cdot \frac{\cancel{7} \cdot 3 \cdot \cancel{k}}{(5k-2)}$$

$$= \boxed{\frac{3(k+1)}{4(5k-2)} \quad \text{or} \quad \frac{3k+3}{20k-8}}$$

33. Graph $4x + 3y = 12$ by first finding the x - and y -intercepts of the equation. Label points.

To find the x -intercept,

set $y = 0$

$$4x + 0 = 12 \Rightarrow 4x = 12 \Rightarrow x = 3$$

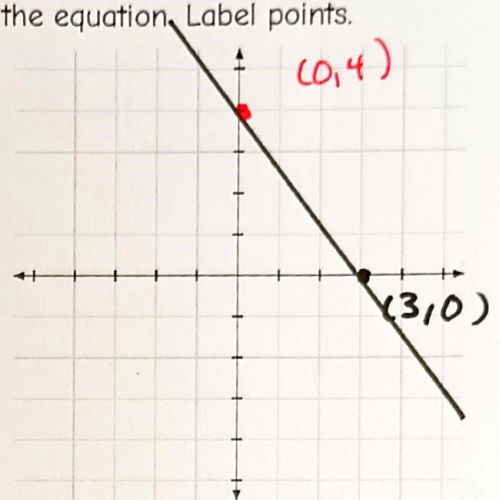
To find the y -intercept,

set $x = 0$

$$0 + 3y = 12 \Rightarrow 3y = 12 \Rightarrow y = 4$$

x -intercept: (3 , 0)

y -intercept: (0 , 4)



34. Graph $x - 2y = 4$ by first writing the equation in slope-intercept form. Label points.

Slope-intercept form : $y = mx + b$

So we need to solve for y .

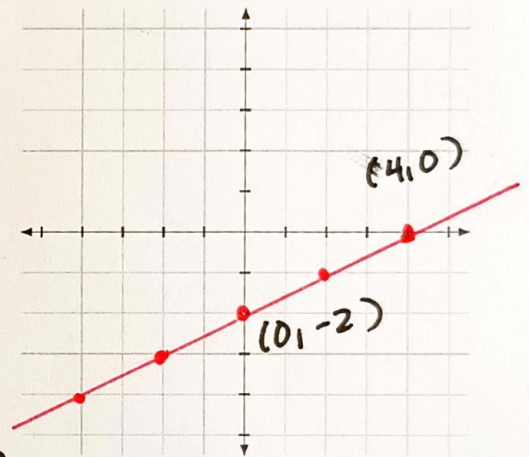
$$x - 2y = 4$$

$$\begin{array}{r} x - 2y = 4 \\ -x \quad -x \end{array}$$

$$\frac{-2y}{-2} = \frac{-x}{-2} + \frac{4}{-2}$$

$$y = \frac{1}{2}x - 2$$

$$m = \frac{1}{2} = \frac{-1}{-2} \text{ and } y\text{-int is } (0, -2)$$



35. Find the solution to the system of linear equations by graphing. If there is no solution or infinitely many solutions state so.

$$\begin{cases} 2x - 14y = -14 \\ 2x + 7y = 28 \end{cases}$$

$$2x - 14y = -14$$

$$\frac{-14y}{-14} = \frac{-2x - 14}{-14} \quad \frac{-14}{-14}$$

$$y = \frac{1}{7}x + 1$$

y -int (0, 1)

$$m = \frac{1}{7}$$

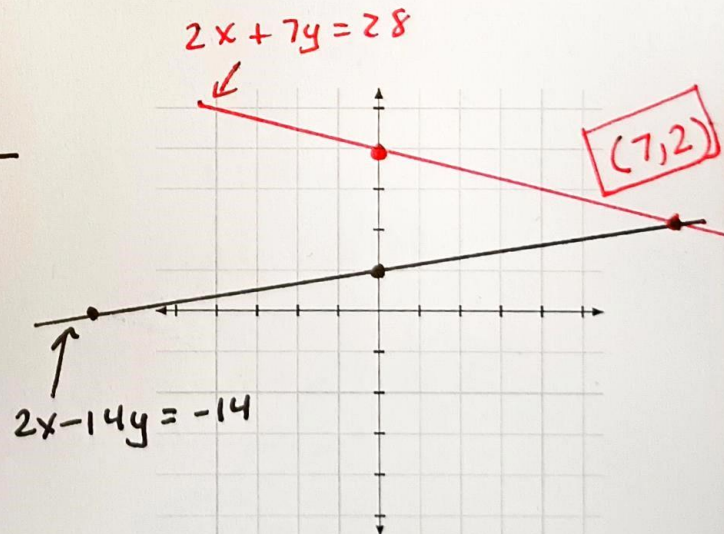
$$2x + 7y = 28$$

$$\frac{7y}{7} = \frac{-2x + 28}{7} \quad \frac{28}{7}$$

$$y = \frac{-2}{7}x + 4$$

y -int (0, 4)

$$m = \frac{-2}{7}$$



36. Nancy bought 7 pounds of oranges and 3 pounds of bananas for \$17. Her husband later bought 3 pounds of oranges and 6 pounds of bananas for \$12. What was the cost per pound of the oranges and the bananas?

Let $x = \text{cost/lb of oranges}$
 $y = \text{cost/lb of bananas}$

Nancy: $7x + 3y = 17$

Husband: $\frac{3x}{3} + \frac{6y}{3} = \frac{12}{3} \Rightarrow x + 2y = 4 \Rightarrow x = 4 - 2y$

$7(4 - 2y) + 3y = 17 \Rightarrow 28 - 14y + 3y = 17$

$\Rightarrow 28 - 11y = 17 \Rightarrow -11y = -11 \Rightarrow y = 1$

$x = 4 - 2(1) = 2$

Oranges cost \$2/lb, bananas cost \$1/lb

37. One serving of trail mix has 67 grams of carbohydrates, which is 22% of the recommended daily amount. What is the total recommended daily amount of carbohydrates? Round to the nearest gram.

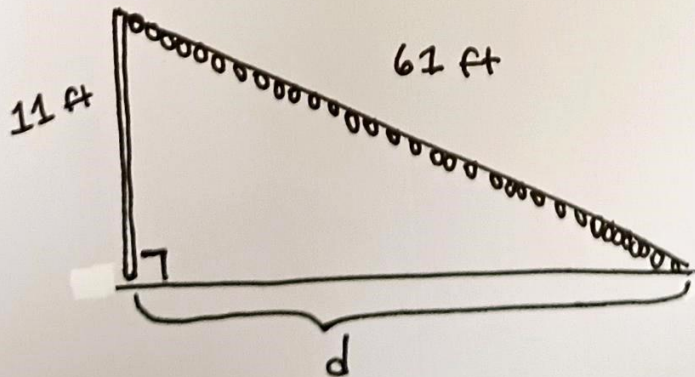
$\frac{67}{.22} = \frac{.22x}{.22}$

where $x = \text{daily recommended amount of carbs}$

$x = 304.54 \text{ grams, so}$

$x \approx 305 \text{ grams}$

38. A 61-foot string of lights will be attached to the top of a 11-foot pole for a holiday display. How far from the base of the pole should the end of the string of lights be anchored?



$d^2 + 11^2 = 61^2$

$d^2 + 121 = 3721$

$d^2 = 3600$

$d = 60 \text{ ft}$

39. An acorn falls straight down from the top of a tall oak tree and hits the ground without hitting any branches on the way down. Solve the equation $-16t^2 + 64 = 0$ for t to find how long it takes to reach the ground.

$$-16t^2 + 64 = 0 \Rightarrow -16(t^2 - 4) = 0$$

$$-16(t+2)(t-2) = 0$$

Use ZPP: ~~$-16 = 0$~~ or $t+2=0$ or $t-2=0$
 ~~$t = -2$~~ or $t = 2$

It takes 2 seconds to reach the ground.

40. At the end of spring break, Lucy left the beach and drove back towards home, driving at a rate of 40 mph. Lucy's friend left the beach for home 30 minutes (half an hour) later, and drove 50 mph. How long did it take Lucy's friend to catch up to Lucy?

	Distance = Rate x Time		
Lucy	$40(t + \frac{1}{2})$	40	$t + \frac{1}{2}$
Lucy's friend	$50t$	50	t

For Lucy's friend to catch up to Lucy, their distances traveled must be the same:

$$40(t + \frac{1}{2}) = 50t \Rightarrow$$

$$40t + 20 = 50t \Rightarrow$$

$$\frac{20}{10} = \frac{10t}{10}$$

$t = 2 \text{ hrs}$

It takes Lucy's friend 2 hrs to catch up to her.